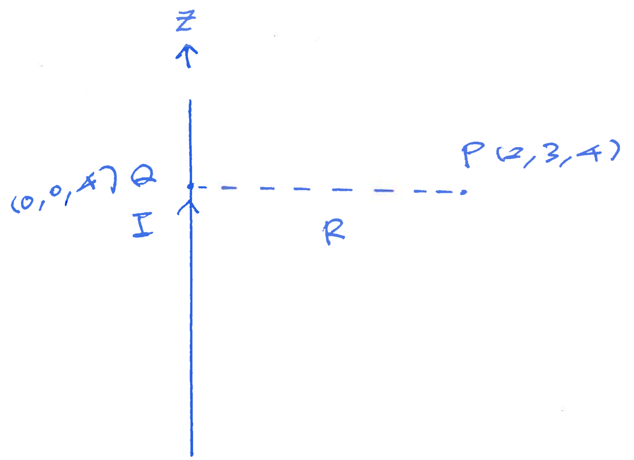
전자기학 7장 연습문제

7.1

(a)



$I = 8 \times 10^{-3}$ (A)

$$R = \sqrt{2^2 + 3^2} = \sqrt{13}$$

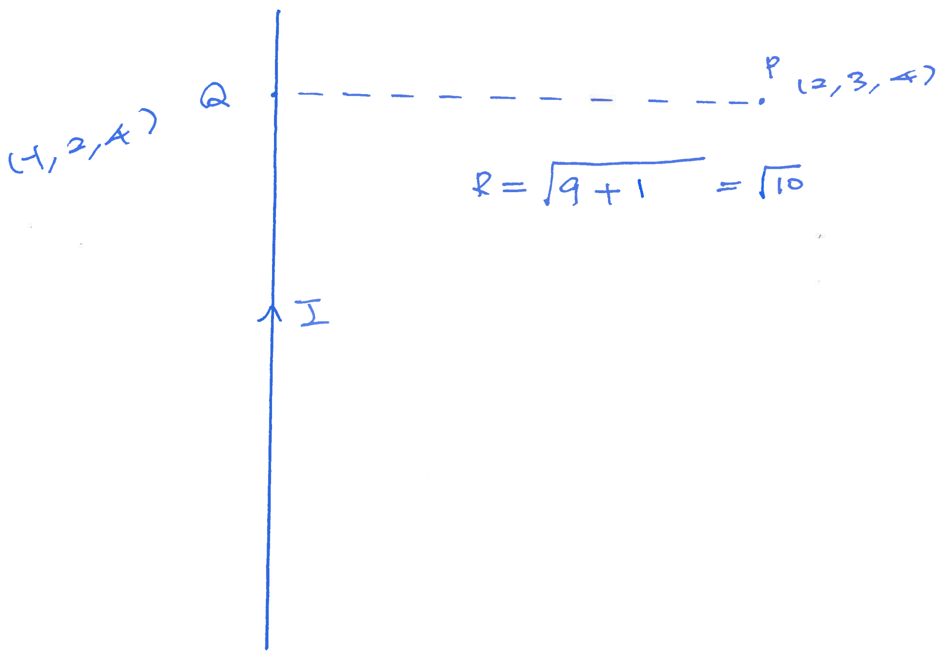
$$\vec{H} = \frac{I}{2\pi R}\hat{\phi} \quad - ①$$

$$\hat{\phi} = \hat{z} \times \frac{\overrightarrow{QP}}{R} = \frac{1}{\sqrt{13}}\hat{z} \times (2\hat{x} + 3\hat{y}) = \frac{1}{\sqrt{13}}(2\hat{y} - 3\hat{x}) \quad - ②$$

$$\vec{H} = \frac{8 \times 10^{-3}}{2\pi\sqrt{13}} \frac{1}{\sqrt{13}}(2\hat{y} - 3\hat{x}) = -0.294 \times 10^{-3}\hat{x} + 0.196 \times 10^{-3}\hat{y} \quad \text{(A/m)}$$

$$= -294\hat{x} + 196\hat{y} \quad (\mu\text{A/m})$$

(b)



$I = 8 \times 10^{-3}$

$R = \sqrt{9+1} = \sqrt{10}$

$\vec{H} = \frac{I}{2\pi R} \hat{\phi}$ — ①

$\hat{\phi} = \hat{z} \times \frac{\overrightarrow{QP}}{R} = \frac{1}{R} \hat{z} \times (3\hat{x} + \hat{y}) = \frac{1}{R}(3\hat{y} - \hat{x})$ — ②

② → ①

$\vec{H} = \frac{I}{2\pi R^2}(-\hat{x} + 3\hat{y}) = \frac{8 \times 10^{-3}}{20\pi}(-\hat{x} + 3\hat{y}) = -0.127 \times 10^{-3}\,\hat{x} + 0.382 \times 10^{-3}\,\hat{y}$ (A/m)

$= -127\,\hat{x} + 382\,\hat{y}$ (μA/m)

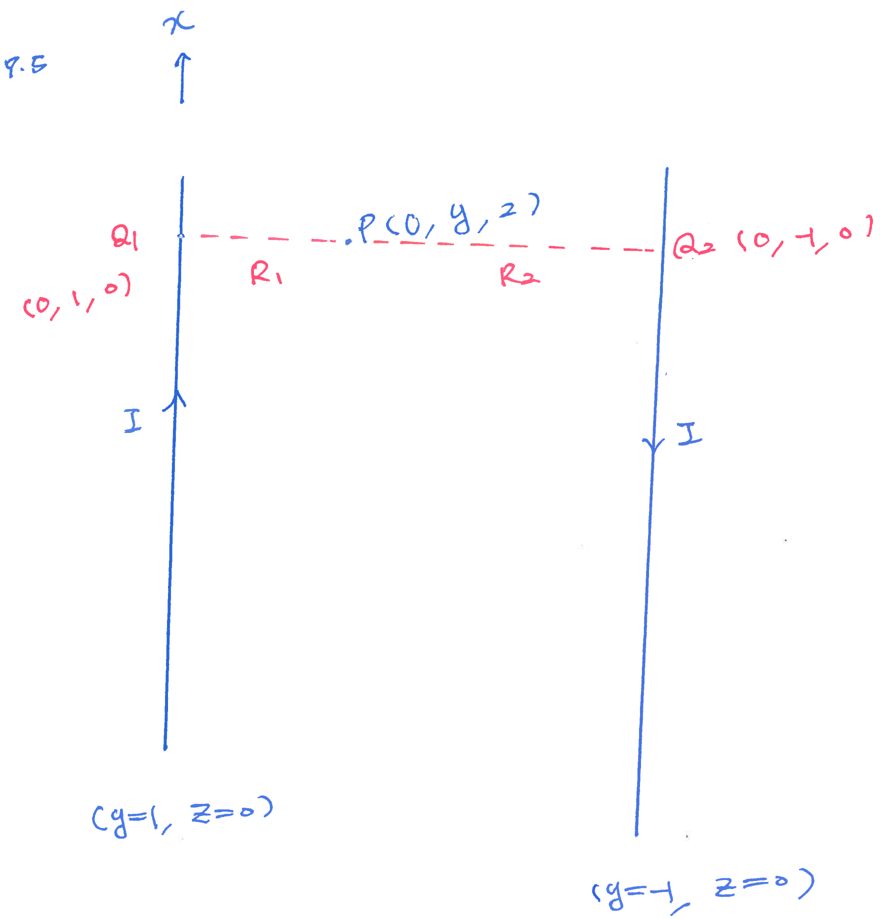
(c)

$$\vec{H} = \vec{H_1} + \vec{H_2}$$

$$= -(127 + 294)\hat{x} + (196 + 382)\hat{y} \quad (\mu A/m)$$

$$= -421\hat{x} + 578\hat{y} \quad (\mu A/m)$$

7.5



$$I = 1\ (A)$$

$$R_1 = \sqrt{(y-1)^2 + 4}$$

$$R_2 = \sqrt{(y+1)^2 + 4}$$

$$\vec{H}_1 = \frac{I}{2\pi R_1}\hat{\phi}_1 \quad -①$$

$$\vec{H}_2 = \frac{I}{2\pi R_2}\hat{\phi}_2 \quad -②$$

$$\hat{\phi}_1 = \hat{x} \times \frac{\overrightarrow{Q_1P}}{R_1} = \frac{1}{R_1}\hat{x} \times [(y-1)\hat{y} + 2\hat{z}]$$

$$= \frac{1}{R_1}[(y-1)\hat{z} - 2\hat{y}] \quad -③$$

③ → ①

$$\vec{H}_1 = \frac{I}{2\pi R_1}\frac{1}{R_1}[-2\hat{y} + (y-1)\hat{z}] = \frac{I}{2\pi R_1^2}[-2\hat{y} + (y-1)\hat{z}] \quad -④$$

$$\hat{\phi}_2 = -\hat{x} \times \frac{\overrightarrow{Q_2P}}{R_2} = -\frac{1}{R_2}\hat{x} \times [(y+1)\hat{y} + 2\hat{z}] = -\frac{1}{R_2}[-2\hat{y} + (y+1)\hat{z}] \quad -⑤$$

⑤ → ②

$$\vec{H}_2 = -\frac{I}{2\pi R_2^2}[-2\hat{y} + (y+1)\hat{z}] \quad -⑥$$

$$\vec{H} = \vec{H}_1 + \vec{H}_2$$

$$= \hat{y}\left(-\frac{I}{\pi R_1^2} + \frac{I}{\pi R_2^2}\right) + \hat{z}\left(\frac{I(y-1)}{2\pi R_1^2} - \frac{I(y+1)}{2\pi R_2^2}\right)$$

$$|\vec{H}| = \sqrt{\left(-\frac{I}{\pi R_1^2} + \frac{I}{\pi R_2^2}\right)^2 + \left(\frac{I(y-1)}{2\pi R_1^2} - \frac{I(y+1)}{2\pi R_2^2}\right)^2}$$

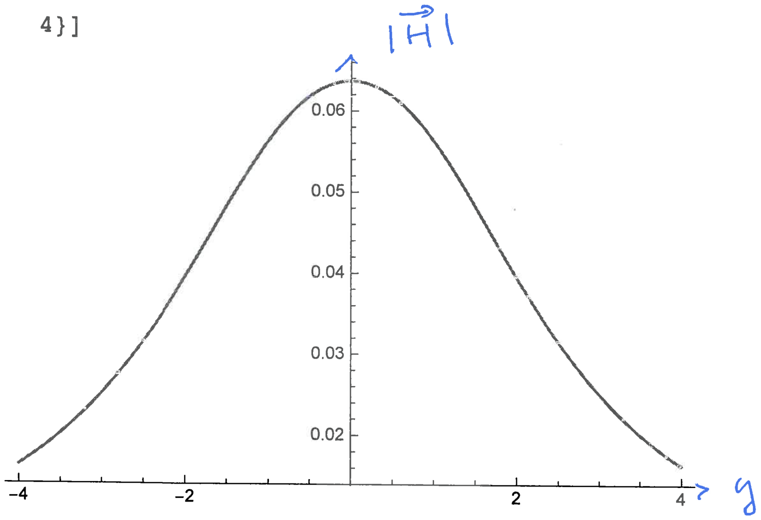
$$\left(\begin{array}{l} I = 1\ (A) \\ R_1^2 = (y-1)^2 + 4 \\ R_2^2 = (y+1)^2 + 4 \end{array}\right)$$

$$= \sqrt{\frac{I^2}{\pi^2}\left[\frac{1}{(y-1)^2+4} - \frac{1}{(y+1)^2+4}\right]^2 + \frac{I^2}{4\pi^2}\left[\frac{y-1}{(y-1)^2+4} - \frac{y+1}{(y+1)^2+4}\right]^2}$$

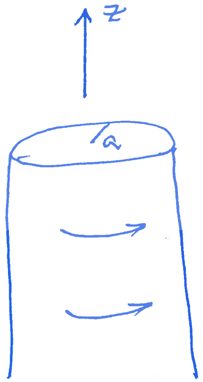
$$= \frac{I}{2\pi}\sqrt{4\left[\frac{1}{y^2-2y+5} - \frac{1}{y^2+2y+5}\right]^2 + \left[\frac{y-1}{y^2-2y+5} - \frac{y+1}{y^2+2y+5}\right]^2}$$

```
In[90]:= h[y_] := (1 / (2 Pi)) Sqrt[4 (1 / (y^2 - 2 y + 5) - 1 / (y^2 + 2 y + 5))^2 +
             ((y - 1) / (y^2 - 2 y + 5) - (y + 1) / (y^2 + 2 y + 5))^2];
         Plot[
          h[
           y],
          {y,
           -4,
           4}]
```

Out[91]=



7.13



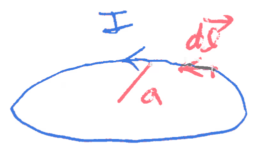
(a) $\vec{H}$ 가 ϕ 와 z 와 무관 함을 보여라

P점의 위치를 ϕ 나 z 를 변화 시켜도

물리적 상황이 변치 않으므로 $\vec{H}$ 는 ϕ 와 z 와 무관함.

(b) 다음과 같은 환전류 문제를 생각하자

$\cdot\, P(x, y, z)$



$(a\cos\phi,\ a\sin\phi,\ 0)$

(xy 평면)

$$d\vec{l} = a\,d\phi\,\hat{\phi} = a\,d\phi\,(-\sin\phi\,\hat{x} + \cos\phi\,\hat{y})$$

$$r = \sqrt{(x-a\cos\phi)^2 + (y-\sin\phi)^2 + z^2}$$

$$\hat{r} = \frac{1}{r}\left[(x-a\cos\phi)\hat{x} + (y-a\sin\phi)\hat{y} + z\hat{z}\right]$$

$d\vec{l} \times \hat{r}$

$$= \frac{a\,d\phi}{r} \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ -\sin\phi & \cos\phi & 0 \\ x - a\cos\phi & y - a\sin\phi & z \end{vmatrix}$$

$$z(\cos\phi\,\hat{x} + \sin\phi\,\hat{y}) + (-x\cos\phi - y\sin\phi + a)\,\hat{z}$$

$$= z\,\hat{\rho} + (-x\cos\phi - y\sin\phi + a)\,\hat{z}$$

$$\Rightarrow \vec{H} = \int_c \frac{I\,d\vec{l} \times \hat{r}}{4\pi r^2}$$

$$= \int \frac{a\,d\phi}{4\pi r^3}\left[z\,\hat{\rho} + (-x\cos\phi - y\sin\phi + a)\,\hat{z}\right]$$

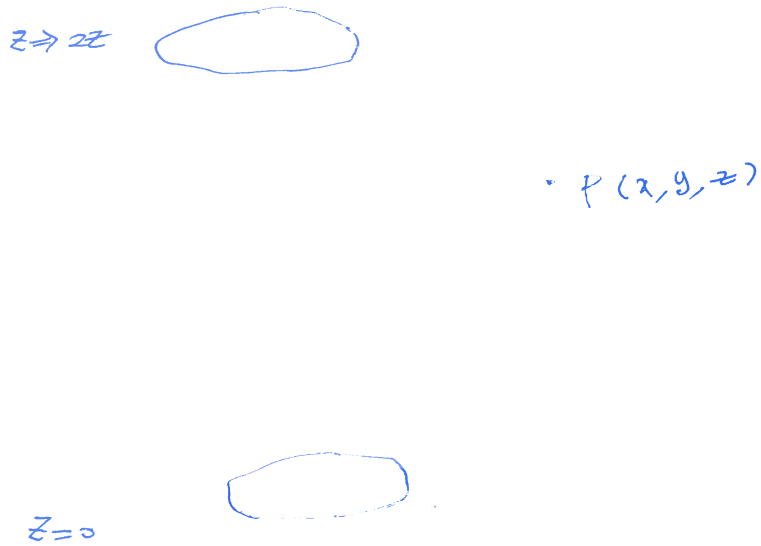
$$= H_\rho\,\hat{\rho} + H_z\,\hat{z}$$

$$H_\rho = \int \frac{az}{4\pi r^3}\,d\phi$$

$$H_z = \int \frac{a(-x\cos\phi - y\sin\phi + a)}{4\pi r^3}\,d\phi$$

$\therefore\ H_\phi = 0$

같은 거리에 있는 두개의 환 전류가 P에 주는 자계 세기는 생각하자

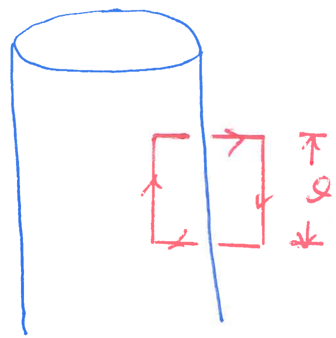


Then $H_\rho = \int \frac{az}{4\pi r^3}\, d\phi + \int \frac{a(-z)}{4\pi r^3}\, d\phi = 0$

$\Rightarrow\ H_\rho = 0$

$\Rightarrow\ \underline{H_\phi = 0,\ H_\rho = 0}$ $\Rightarrow\ \underline{\vec{H} = H(\rho)\,\hat{z}}$

(d)



Ampere line

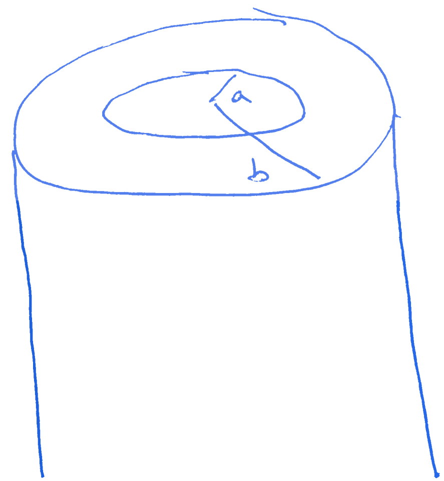
$$\oint_C \vec{H}\cdot d\vec{l} = H\,l$$

$$I_{inside} = K_a\, l$$

$$H = K_a$$

$$\Rightarrow \quad \vec{H} = \begin{cases} K_a\,\hat{z} & \rho < a \\ 0 & \rho > a \end{cases}$$

(e)



$\rho < a \qquad \vec{H} = (K_a + K_b)\hat{z}$

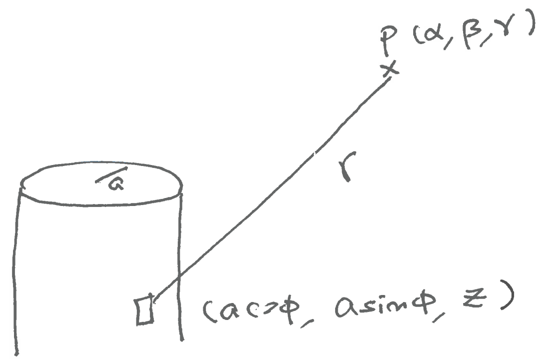
$a < \rho < b \qquad \vec{H} = K_b\hat{z}$

$b < \rho \qquad \vec{H} = 0$

위 문제는 면적분으로 바로 구할 수도 있다

$$\vec{H} = \int_S \frac{\vec{K} \times \hat{r}}{4\pi r^2} \, ds$$

* Residue Theorem 연습!!



$$r = \sqrt{(a\cos\phi - \alpha)^2 + (a\sin\phi - \beta)^2 + (z-\gamma)^2}$$

$$\hat{r} = \frac{1}{r}\left[(\alpha - a\cos\phi)\hat{x} + (\beta - a\sin\phi)\hat{y} + (\gamma - z)\hat{z} \right]$$

$$\vec{K} = K_a \hat{\phi} = K_a(-\sin\phi \, \hat{x} + \cos\phi \, \hat{y})$$

$$\vec{K} \times \hat{r}$$

$$= \frac{1}{r} \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ -Ka\sin\phi & Ka\cos\phi & 0 \\ \alpha - a\cos\phi & \beta - a\sin\phi & \gamma - z \end{vmatrix}$$

$$= \frac{1}{r} \Big[Ka\cos\phi\,(\gamma - z)\,\hat{x} + Ka(\sin\phi)\,(\gamma - z)\,\hat{y}$$
$$+ \hat{z} \underbrace{\left[-Ka\sin\phi\,(\beta - a\sin\phi) - Ka\cos\phi\,(\alpha - a\cos\phi) \right]}_{Ka\,(a - \alpha\cos\phi - \beta\sin\phi)} \Big]$$

$$= \frac{Ka}{r} \left[(\gamma - z)\cos\phi\,\hat{x} + (\gamma - z)\sin\phi\,\hat{y} + (a - \alpha\cos\phi - \beta\sin\phi)\,\hat{z} \right]$$

$$ds = a\,d\phi\,dz$$

$$\vec{H} = \int \frac{K_a}{4\pi r^3}\left[(\gamma - z)\cos\phi\,\hat{x} + (\gamma - z)\sin\phi\,\hat{y} + (a - \alpha\cos\phi - \beta\sin\phi)\,\hat{z} \right] a\,d\phi\,dz$$

$$= H_x\,\hat{x} + H_y\,\hat{y} + H_z\,\hat{z} \qquad -①$$

$$\left.\begin{aligned} H_x &= \frac{aK_a}{4\pi}\int \frac{(\gamma - z)\cos\phi}{r^3}\,d\phi\,dz \\ H_y &= \frac{aK_a}{4\pi}\int \frac{(\gamma - z)\sin\phi}{r^3}\,d\phi\,dz \\ H_z &= \frac{aK_a}{4\pi}\int \frac{a - \alpha\cos\phi - \beta\sin\phi}{r^3}\,d\phi\,dz \end{aligned}\right\} \qquad -②$$

(i) H_x 계산

$$H_x = \frac{aK_a}{4\pi}\int_{-\infty}^{\infty} dz \int_0^{2\pi} d\phi \frac{(\gamma - z)\cos\phi}{[(a\cos\phi-\alpha)^2 + (a\sin\phi-\beta)^2 + (z-\gamma)^2]^{\frac{3}{2}}}$$

$$= \frac{aK_a}{4\pi}\int_0^{2\pi} d\phi \cos\phi \; J(\phi) \qquad - ③$$

where

$$J(\phi) = \int_{-\infty}^{\infty} dz \frac{\gamma - z}{[(a\cos\phi-\alpha)^2 + (a\sin\phi-\beta)^2 + (z-\gamma)^2]^{\frac{3}{2}}} \qquad \left(\begin{array}{c} z-\gamma \equiv u \\ dz = du \end{array} \right)$$

$$= \int_{-\infty}^{\infty} du \frac{-u}{[(a\cos\phi-\alpha)^2 + (a\sin\phi-\beta)^2 + u^2]^{\frac{3}{2}}}$$

$$= 0 \qquad (\because \text{ odd function}) \qquad - ④$$

(ii) H_y 계산

$$H_y = \frac{aK_a}{4\pi}\int_{-\infty}^{\infty} dz \int_0^{2\pi} d\phi \frac{(\gamma - z)\sin\phi}{\left[(a\cos\phi-\alpha)^2+(a\sin\phi-\beta)^2+(z-\gamma)^2\right]^{\frac{3}{2}}}$$

$$= \frac{aK_a}{4\pi}\int_0^{2\pi} d\phi \sin\phi \; J(\phi)$$

$$= 0$$

(iii) H_z 계산

$$H_z = \frac{aK_a}{4\pi}\int_{-\infty}^{\infty} dz \int_0^{2\pi} d\phi \frac{a - \alpha\cos\phi - \beta\sin\phi}{[(a\cos\phi-\alpha)^2 + (a\sin\phi-\beta)^2 + (z-\gamma)^2]^{\frac{3}{2}}}$$

$$= \frac{aK_a}{4\pi}\int_0^{2\pi} d\phi\,(a - \alpha\cos\phi - \beta\sin\phi)\;K(\phi) \qquad \text{— } 5$$

$$K(\phi) = \int_{-\infty}^{\infty} dz \frac{1}{[(a\cos\phi-\alpha)^2 + (a\sin\phi-\beta)^2 + (z-\gamma)^2]^{\frac{3}{2}}} \qquad \left(\begin{array}{l} u = z-\gamma \\ du = dz \end{array}\right)$$

$$= \int_{-\infty}^{\infty} du \frac{1}{[(a\cos\phi-\alpha)^2 + (a\sin\phi-\beta)^2 + u^2]^{\frac{3}{2}}}$$

$$= 2\int_0^{\infty} du \frac{1}{[(a\cos\phi-\alpha)^2 + (a\sin\phi-\beta)^2 + u^2]^{\frac{3}{2}}}$$

$$= 2\int_0^{\infty} \frac{du}{(u^2 + c^2)^{\frac{3}{2}}} \qquad \left(c^2 = (a\cos\phi-\alpha)^2 + (a\sin\phi-\beta)^2\right)$$

Put

$$u = c \tan\theta$$

$$u^2 + c^2 = c^2(1+\tan^2\theta) = c^2 \sec^2\theta$$

$$du = c \sec^2\theta \, d\theta$$

$$\Rightarrow \quad K(\phi) = 2\int_0^{\frac{\pi}{2}} \frac{c \sec^2\theta}{c^3 \sec^3\theta} \, d\theta$$

$$= \frac{2}{c^2}\int_0^{\frac{\pi}{2}} \cos\theta \, d\theta$$

$$= \frac{2}{c^2} \sin\theta \Big|_{\theta=0}^{\theta=\frac{\pi}{2}}$$

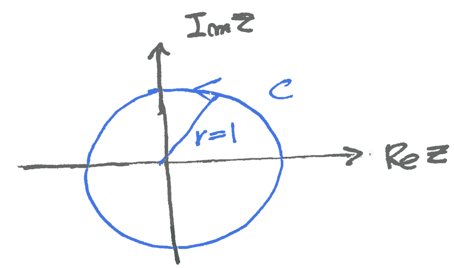
$$= \frac{2}{c^2} = \frac{2}{(a\cos\phi-\alpha)^2 + (a\sin\phi-\beta)^2} \qquad - (6)$$

④ → ⑤

$$H_z = \frac{aK_a}{2\pi}\int_0^{2\pi} d\phi \; \frac{a-\alpha\cos\phi-\beta\sin\phi}{(a\cos\phi-\alpha)^2+(a\sin\phi-\beta)^2}$$

$$= \frac{aK_a}{2\pi}\int_0^{2\pi} d\phi \; \frac{a-\alpha\cos\phi-\beta\sin\phi}{(a^2+\alpha^2+\beta^2)-2a\alpha\cos\phi-2a\beta\sin\phi}$$

$$\left(\begin{array}{l} z = e^{i\phi} \\ dz = ie^{i\phi}d\phi = iz\,d\phi \end{array}\right)$$



$$= \frac{aK_a}{2\pi}\oint_C \frac{dz}{iz} \; \frac{a-\alpha\dfrac{z+z^{-1}}{2}-\beta\dfrac{z-z^{-1}}{2i}}{(a^2+\alpha^2+\beta^2)-\cancel{2}a\alpha\dfrac{z+z^{-1}}{\cancel{2}}-\cancel{2}a\beta\dfrac{z-z^{-1}}{\cancel{2}i}}$$

$$= \frac{aK_a}{2\pi}\oint_C \frac{dz}{iz} \; \frac{a+\frac{z}{2}(-\alpha+i\beta)+\frac{z^{-1}}{2}(-\alpha-i\beta)}{(a^2+\alpha^2+\beta^2)+z(-a\alpha+ia\beta)+z^{-1}(-a\alpha-ia\beta)}$$

$$= \frac{aK_a}{2\pi}\oint_C \frac{dz}{iz} \; \frac{a-\dfrac{\alpha-i\beta}{2}z-\dfrac{\alpha+i\beta}{2}z^{-1}}{(a^2+\alpha^2+\beta^2)-a(\alpha-i\beta)z-a(\alpha+i\beta)z^{-1}}$$

$$= \frac{aK_a}{4\pi}\oint_C \frac{dz}{iz} \; \frac{2a-(\alpha-i\beta)z-(\alpha+i\beta)z^{-1}}{(a^2+\alpha^2+\beta^2)-a(\alpha-i\beta)z-a(\alpha+i\beta)z^{-1}}$$

$$= \frac{aK_a}{4\pi i}\oint_C dz \; \frac{\dfrac{2az-(\alpha-i\beta)z^2-(\alpha+i\beta)}{z}}{-a(\alpha-i\beta)z^2+(a^2+\alpha^2+\beta^2)z-a(\alpha+i\beta)}$$

$$= \frac{aKa}{4\pi i} \oint_c dz \, \frac{(\alpha - i\beta) z^2 - 2az + (\alpha + i\beta)}{z\left[a(\alpha - i\beta) z^2 - (a^2 + \alpha^2 + \beta^2) z + a(\alpha + i\beta)\right]} \quad - ①$$

pole:

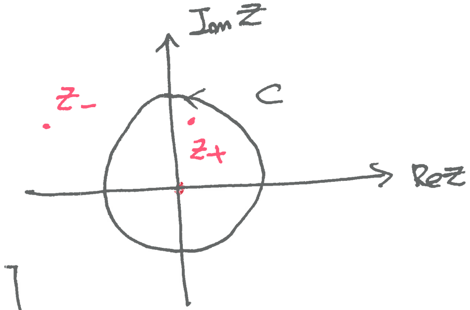
$$\left(\begin{array}{l} z = 0 \\[2ex] z_+ = \dfrac{a}{\alpha - i\beta} \\[2ex] z_- = \dfrac{\alpha + i\beta}{a} \end{array} \right) \quad - ②$$

$$H_z = \frac{\cancel{a} Ka}{4\pi i \, \cancel{a} (\alpha - i\beta)} \oint_c \frac{(\alpha - i\beta) z^2 - 2az + (\alpha + i\beta)}{z (z - z_+)(z - z_-)}$$

$$= \frac{Ka}{4\pi i (\alpha - i\beta)} \oint_c dz \, \frac{(\alpha - i\beta) z^2 - 2az + (\alpha + i\beta)}{z (z - z_+)(z - z_-)} \quad - ③$$

(i) $\alpha^2+\beta^2 \geq a^2$

$|z_+| < 1$, $|z_-| > 1$



$$H_z = \frac{ka}{2(\alpha-i\beta)}\left[\frac{\alpha+i\beta}{z_+z_-} + \frac{(\alpha-i\beta)z_+^2 - 2az_+ + (\alpha+i\beta)}{z_+(z_+-z_-)}\right]$$

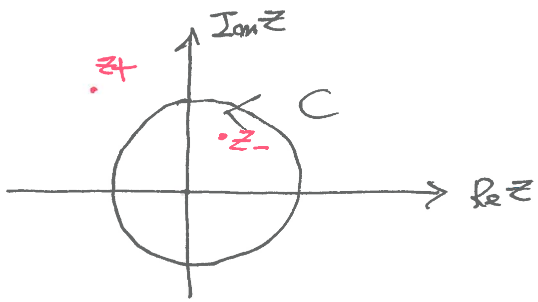
$$= \frac{ka}{2(\alpha-i\beta)}\left[\frac{\alpha+i\beta}{\frac{\alpha+i\beta}{\alpha-i\beta}} + \frac{\boxed{(\alpha-i\beta)\frac{a^2}{(\alpha-i\beta)^2} - 2a\frac{a}{\alpha-i\beta} + (\alpha+i\beta)}}{\boxed{\frac{a}{\alpha-i\beta}\left[\frac{a}{\alpha-i\beta} - \frac{\alpha+i\beta}{a}\right]}}\right]$$

$$\frac{-a^2+(\alpha^2+\beta^2)}{\alpha-i\beta}$$

$$\frac{1}{(\alpha-i\beta)^2}(a^2-\alpha^2-\beta^2)$$

$$= \frac{ka}{2(\alpha-i\beta)}\left[(\alpha-i\beta)-(\alpha-i\beta)\right] = 0$$

(ii) $\alpha^2+\beta^2 \le a^2$



$|z_+| > 1 \ , \quad |z_-| < 1$

$$H_z = \frac{Ka}{2(\alpha - i\beta)}\left[\frac{\alpha+i\beta}{z_+z_-} + \frac{(\alpha-i\beta)z_-^2 - 2az_- + (\alpha+i\beta)}{z_-(z_- - z_+)}\right]$$

$$= \frac{Ka}{2(\alpha-i\beta)}\left[\frac{\alpha+i\beta}{\underbrace{\frac{\alpha+i\beta}{\alpha-i\beta}}_{\alpha-i\beta}} + \frac{\boxed{(\alpha-i\beta)\frac{(\alpha+i\beta)^2}{a^2} - 2a\frac{\alpha+i\beta}{a} + (\alpha+i\beta)}}{\boxed{\frac{\alpha+i\beta}{a}\left(\frac{\alpha+i\beta}{a} - \frac{a}{\alpha-i\beta}\right)}}\right]$$

$$\frac{(\alpha+i\beta)(\alpha^2+\beta^2-a^2)}{a^2}$$

$$\frac{(\alpha+i\beta)(\alpha^2+\beta^2-a^2)}{a^2(\alpha-i\beta)}$$

$$= \frac{Ka}{2(\alpha-i\beta)} \, 2(\alpha-i\beta)$$

$$= Ka$$

$$\Rightarrow \quad \vec{H} = \begin{cases} K_a \hat{z} & \alpha^2+\beta^2 \leq a^2 \\ 0 & \alpha^2+\beta^2 \geq a^2 \end{cases}$$

$$\Rightarrow \quad \vec{H} = \begin{cases} K_a \hat{z} & \rho < a \\ 0 & \rho > a \end{cases}$$

7.34

(a)

$0 \le \rho \le 6 \qquad H = \dfrac{16}{2\pi\rho}\,\hat{\phi} = \dfrac{8}{\pi\rho}\,\hat{\phi}$

$6 \le \rho \le 10 \qquad H = \dfrac{4}{2\pi\rho}\,\hat{\phi} = \dfrac{2}{\pi\rho}\,\hat{\phi}$

$10 \le \rho \qquad H = 0$

(b)

```
h[x_] := If[0 ≤ x ≤ 6, 8 / (Pi x), If[6 < x ≤ 10, 2 / (Pi x), 0]];
Plot[h[x], {x, 0, 13}]
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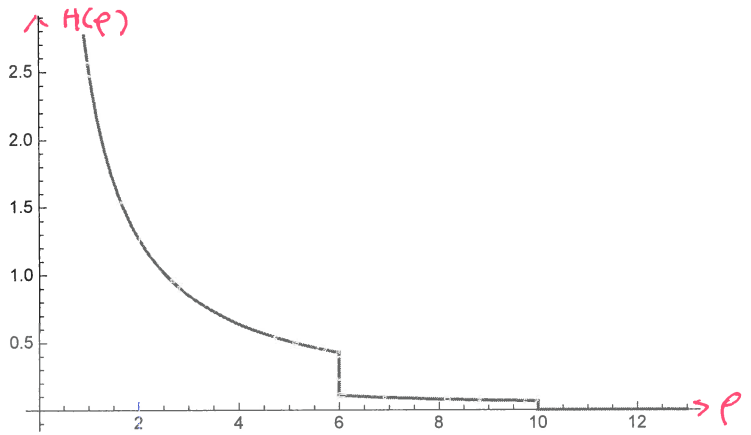

(c) $\Phi_B = \int_S \vec{B} \cdot \hat{u}_N \, ds$

$$\vec{B} = \mu_0 \vec{H} = \begin{cases} \dfrac{\mu_0 8}{\pi \rho} \hat{\phi} & 0 \le \rho \le 6 \\ \dfrac{\mu_0 2}{\pi \rho} \hat{\phi} & 6 \le \rho \le 10 \\ 0 & 10 \le \rho \end{cases}$$

$\hat{u}_N = \hat{\phi}$

$$\Rightarrow \quad \Phi_B = \int_1^7 d\rho \int_0^1 dz \; B$$

$$= \int_1^6 \frac{8\mu_0}{\pi \rho} d\rho + \int_6^7 \frac{2\mu_0}{\pi \rho} d\rho$$

$$= \frac{8\mu_0}{\pi} \ln 6 + \frac{2\mu_0}{\pi} \ln \frac{7}{6}$$

$$= \frac{2\mu_0}{\pi} \left[4 \ln 6 + \ln \frac{7}{6} \right]$$

7.36

$$\vec{A} = (3y - z)\hat{x} + 2xz\,\hat{y}$$

(a) $\vec{\nabla} \cdot \vec{A} = \dfrac{\partial}{\partial x}(3y - z) + \dfrac{\partial}{\partial y}(2xz) = 0$

(b) $P(2, -1, 3)$

$$A(P) = -6\hat{x} + 12\hat{y} \quad (\text{Wb/m})$$

$$\vec{B} = \vec{\nabla} \times \vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ 3y - z & 2xz & 0 \end{vmatrix} = \hat{x}(-2x) + \hat{y}(-1) + \hat{z}(2z - 3)$$

$$= -2x\hat{x} - \hat{y} + (2z - 3)\hat{z}$$

$$\vec{B}(P) = -4\hat{x} - \hat{y} + 3\hat{z} \quad (\text{T})$$

$$\vec{H}(P) = \frac{1}{\mu_0}\vec{B}(P) = \frac{1}{\mu_0}(-4\hat{x} - \hat{y} + 3\hat{z}) \quad (\text{A/m})$$

$$\vec{\nabla} \times \vec{H} = \frac{1}{\mu_0} \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -2x & -1 & 2z-3 \end{vmatrix} = 0$$

$$\vec{J}(P) = 0$$

7.40

$$\Phi_B \equiv \int_S \vec{B} \cdot \hat{u}_N \, ds$$

$$= \int_S (\vec{\nabla} \times \vec{A}) \cdot \hat{u}_N \, ds \qquad (\vec{B} = \vec{\nabla} \times \vec{A})$$

$$= \int_C \vec{A} \cdot d\vec{s} \qquad (\text{Stoke's theorem})$$

7.41

$$\vec{A} = 50\rho^2 \hat{z} \qquad (\mathrm{Wb/m})$$

(a) $\vec{B} = \vec{\nabla} \times \vec{A}$

$$= \frac{1}{\rho} \begin{vmatrix} \hat{\rho} & \rho\hat{\phi} & \hat{z} \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ 0 & 0 & 50\rho^2 \end{vmatrix}$$

$$= \frac{1}{\rho} \left[\rho\hat{\phi} (-100\rho) \right]$$

$$= -100\rho\hat{\phi}$$

$$\vec{H} = \frac{\vec{B}}{\mu_0} = -\frac{100\rho}{\mu_0}\hat{\phi} \qquad (\mathrm{A/m})$$

$$\vec{B} = -100\rho\hat{\phi} \qquad (\mathrm{T})$$

(b)

$$\vec{J} = \vec{\nabla} \times \vec{H}$$

$$= \frac{1}{\rho} \begin{vmatrix} \hat{\rho} & \rho\hat{\phi} & \hat{z} \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ 0 & \left(-\frac{100}{\mu_0}\rho^2\right) & 0 \end{vmatrix}$$

$$= \frac{1}{\rho}\left[\frac{\partial}{\partial \rho}\left(-\frac{100}{\mu_0}\rho^2\right) \hat{z} \right]$$

$$\cancel{= -\frac{100}{\mu_0} \frac{1}{\rho} \hat{z}}$$

$$= -\frac{200}{\mu_0} \hat{z} \quad (A/m^2)$$

(c) $I = \int_S \vec{J} \cdot \hat{a}_N \, dS = \frac{200}{\mu_0} \pi = \frac{200\pi}{4\pi \times 10^{-7}} = 50 \times 10^{7}$ $\cancel{= 5 \times 10^{8} (A) = 5 (MA)}$

$= 500 \times 10^{6} (A) = 500 (MA)$

(d) $\oint_C \vec{H} \cdot d\vec{l} = I_{inside} = 500 (MA)$